

## Philosophy 12: Introduction to Causal Reasoning

### Answers to study questions for Lecture 8: “Conditional Independence”

1. Answer(s): (c), (d)
2. Answer(s): (c), (d)
3. Answer(s): (a), (c)

The reason C is correct is that among the cards {Ace of diamonds, Ace of hearts, King of diamonds, King of hearts}, the frequency of diamonds is  $\frac{2}{4}$ , and the frequency of diamonds conditional on a red card is also  $\frac{2}{4}$ , so picking a diamond is in fact independent of picking a red card in this group.

The reason B is not correct is that the frequency of diamonds in a regular deck is  $\frac{13}{52} = \frac{1}{4}$ . The frequency of diamonds conditional on a red card is  $\frac{13}{26} = \frac{1}{2}$ , so picking a diamond is not independent of picking a red card overall.

4. Answer(s): (b), (c)

We are trying to establishing that smoker is or is not independent of blond hair, *given* that a person is male. Therefore, all our histograms must be conditional on being male. The reason A is not correct is that histogram only represents the frequency of smokers, not smokers among males. The reason D is not correct is that it is a histogram of a frequency that is conditional on being blond, not on being male.

5. Answer(s): (b)

The question is whether smoking is independent of blond hair, given that a person is male. In frequencies, this conditional independence translates to:

$$\text{Fr}(\text{SMOKER} = \text{Yes} \mid \text{SEX} = \text{Male}) = \text{Fr}(\text{SMOKES} = \text{Yes} \mid \text{SEX} = \text{Male} \ \& \ \text{HAIR COLOR} = \text{Blond})$$

. Thus, the independence only holds if histograms B and C are the same, which they are not.

6. Answer(s): (a)

The question is whether smoking is independent of being male, given that a person has blond hair. In frequencies, this conditional independence translates to  $\text{Fr}(\text{SMOKES} = \text{Yes} \mid \text{SEX} = \text{Male}) = \text{Fr}(\text{SMOKES} = \text{Yes} \mid \text{SEX} = \text{Male} \ \& \ \text{HAIR COLOR} = \text{Blond})$ . Thus, the independence only holds if histograms C and D are the same, which they are not.

7. Answer(s):  $\frac{1}{2}$

8. Answer(s):  $\frac{2}{3}$ . Remember,  $\text{Fr}_S(\text{TAKEN BIOLOGY} = \text{Yes} \mid \text{SEX} = \text{Male})$  is the number of individuals who are taking biology and are male, divided by the number of males.

9. Answer(s): 1.0. Remember,  $\text{Fr}_S(\text{TAKING BIOLOGY} = \text{Yes} \mid \text{CLASS} = \text{Senior})$  is the number of individuals who are taking biology and are seniors, divided by the number of seniors.

10. Answer(s): 1.0. Remember,  $\text{Fr}_S(\text{TAKING BIOLOGY} = \text{Yes} \mid \text{SEX} = \text{Male} \ \& \ \text{CLASS} = \text{Senior})$  is the number of individuals who are taking biology and are male and are seniors, divided by the total number of male seniors.

11. Answer(s): (c)

12. Answer(s): (c)

13. Answer(s): (b), (c)

14. Answer(s): (b), (d)

Remember, in order to establish whether SEX and COLLEGE PLANS are independent conditional on PARENTAL ENCOURAGEMENT, we need to establish whether, conditional on each value of PARENTAL ENCOURAGEMENT, each value of SEX is independent of each value of COLLEGE PLANS. Thus, B is correct because

$$\text{Fr}(\text{CP} = \text{Yes} \mid \text{PE} = \text{High}) = \text{Fr}(\text{CP} = \text{Yes} \mid \text{SEX} = \text{Male}, \text{PE} = \text{High})$$

establishes that  $\text{CP} = \text{Yes} \perp\!\!\!\perp \text{SEX} = \text{Male} \mid \text{PE} = \text{High}$ , which is one of the conditions we are interested in. Similarly, D is correct because

$$\text{Fr}(\text{CP} = \text{Yes} \mid \text{PE} = \text{High}) = \text{Fr}(\text{CP} = \text{Yes} \mid \text{SEX} = \text{Female}, \text{PE} = \text{High})$$

establishes that  $\text{CP} = \text{Yes} \perp\!\!\!\perp \text{SEX} = \text{Female} \mid \text{PE} = \text{High}$ , which is one of the conditions we are interested in. The reason A is not correct is that

$$\text{Fr}(\text{CP} = \text{Yes} \mid \text{SEX} = \text{Male}) = \text{Fr}(\text{CP} = \text{Yes} \mid \text{SEX} = \text{Male}, \text{PE} = \text{High})$$

establishes that  $\text{CP} = \text{Yes} \perp\!\!\!\perp \text{PE} = \text{High} \mid \text{SEX} = \text{Male}$ , which is not what we're interested in. The reason C is not correct is that

$$\text{Fr}(\text{PE} = \text{High} \mid \text{SEX} = \text{Male}) = \text{Fr}(\text{PE} = \text{High} \mid \text{SEX} = \text{Male}, \text{CP} = \text{Yes})$$

establishes that  $\text{PE} = \text{High} \perp\!\!\!\perp \text{CP} = \text{Yes} \mid \text{SEX} = \text{Male}$ , which is not what we're interested in. The reason E is not correct is that

$$\text{Fr}(\text{PE} = \text{High} \mid \text{SEX} = \text{Female}) = \text{Fr}(\text{PE} = \text{High} \mid \text{SEX} = \text{Female}, \text{CP} = \text{Yes})$$

establishes that  $\text{PE} = \text{High} \perp\!\!\!\perp \text{CP} = \text{Yes} \mid \text{SEX} = \text{Female}$ , which is not what we're interested in.

15. Answer(s): (b), (c)

Remember,  $\text{Fr}(\text{CP} = \text{Yes} \mid \text{PE} = \text{High})$  equals the number of people with both  $\text{CP} = \text{Yes}$  and  $\text{PE} = \text{High}$ , divided by the number with  $\text{PE} = \text{High}$ .

16. Answer(s): (b), (d)

Remember,  $\text{Fr}(\text{CP} = \text{Yes} \mid \text{PE} = \text{High}, \text{SEX} = \text{Male})$  equals the number of people with  $\text{CP} = \text{Yes}$ ,  $\text{PE} = \text{High}$ , and  $\text{SEX} = \text{Male}$  divided by the number with  $\text{PE} = \text{High}$  and  $\text{SEX} = \text{Male}$ .